Djumabayev N.
Ajiniyoz nomidagi Nukus davlat
pedagogika instituti dotsenti

MATEMATIK INDUKSIYA METODIDAN FOYDALANIB AYNIYATLARNI ISBOTLASH

Annotatsiya. *Ushbu maqolada o'quvchilarning matematika faniga qiziqishlarini yanada orttiradigan misol va masalalar berilgan. Mazkur misol va masalalar ularning bilim va ko'nikmalarini shakllantirishda katta ahamiyatga ega. Matematika faniga qiziqadigan o'quvchilarning qiziqishlarini yanada orttirish, mantiqiy masalalarni echishga o'rgatish katta ahamiyatga ega.*

Tayanch so'zlar: *Matematik induktsiya metodi, tenglikni isbotlash, gipoteza, tastıyıqlaw n qálegen natural san bolg'anda durıs bo'ladı.*

ДОКАЗАТЕЛЬСТВО ТОЖДЕСТВ МЕТОДОМ МАТЕМАТИЧЕСКОЙ ИНДУКЦИИ

Аннотация. *В этой статье приведены примеры и задачи, которые еще больше повысят интерес учащихся к математике. Эти примеры и задачи имеют большое значение в формировании их знаний и навыков. Большое значение имеет дальнейшее повышение интереса учащихся, интересующихся математикой, обучение их решению логических задач.*

Ключевые слова: *Метод математической индукции, доказательство равенства, гипотеза, утверждение верны, когда n - любое натуральное число.*

PROVING IDENTITIES USING THE METHOD OF MATHEMATICAL INDUCTION

Annotation. *This article provides examples and problems that will increase students' interest in mathematics. These examples and tasks are of great importance in the formation of their knowledge and skills. It is very important to increase the interest of students interested in mathematics and teach them to solve logical problems.*

Key words: *The method of mathematical induction, proof of equality, hypothesis, and statement are valid when n is any natural number.*

KIRISH.

Matematikaning eng muhim tushunchalaridan biri bu – isbotlashdir. Har qanday matematik qoida, qonuniyat yoki formula ilmiy asosda dalillanishi zarur. Shunday usullardan biri matematik induksiya metodi bo'lib, u ko'plab formulalar va ayniyatlarni isbotlashda keng qo'llaniladi.

Matematik induksiya metodi yordamida tabiiy sonlar ketma-ketligida berilgan mulohazalarning to'g'riligini isbotlash mumkin. Bu usul uch bosqichdan iborat: birinchi bosqichda mulohazaning eng kichik son uchun to'g'riligini ko'rsatish, ikkinchi bosqichda esa agar u ma'lum bir son uchun to'g'ri bo'lsa, undan keyingi son uchun ham to'g'ri ekanligini isbotlash talab etiladi. Shu yo'l bilan mulohaza barcha natural sonlar uchun to'g'ri ekanligi aniqlanadi.

TADQIQOT MATERIALLARI VA USULLARI

Matematik induksiya metodining asosi quyidagi prinsipga tayanadi:

Agar biror tasdiqlash

1). n=1 bo‘lganda to‘g‘ri va

2). ushbu tasdiqning qandaydir erkli n=k (k≥1) manosida tog‘riligiga uning n=k+1 bo‘lganda ham to‘g‘ri bo‘lishi kelib chiqsa, unda u tasdiq istalgan natural n soni uchun to‘g‘ri bo‘ladi.

Shu prinsipga asoslangan isbotlash metodi matematik induksiya metodi deb ataladi.

Matematik induksiya metodidan foydalanib ayniyatlarni isbotlashni ko‘rib chiqamiz:

1-mısal. Ayniyatni isbotlang:

$$S_n=1^2+3^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1) \qquad (1)$$

Yechilishi. 1. n=1 bo‘lsa gipoteza to‘g‘ri bo‘ladi, chunki

$$S_1=1^2=\frac{1}{3}\cdot 1\cdot 1\cdot 3$$

2.n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaymiz, ya’ni

$$S_\kappa=1^2+3^2+\ldots+(2\kappa-1)^2=\frac{1}{3}\kappa(2\kappa-1)(2\kappa+1)$$

(1) tasdiqlashning n=k+1 uchun yani,

$$S_{\kappa+1}=1^2+3^2+\ldots+(2\kappa-1)^2+(2\kappa+1)^2=\frac{1}{3}(\kappa+1)(2\kappa+1)(2\kappa+3)$$

tengligining to‘g‘riligini isbotlaymiz:

$$S_{\kappa+1}=1^2+3^2+\ldots+(2\kappa-1)^2+(2\kappa+1)^2=S_\kappa+(2\kappa+1)^2=\frac{1}{3}\kappa(2\kappa-1)(2\kappa+1)+(2\kappa+1)^2=$$

$$=(2\kappa+1)\left(\frac{1}{3}\kappa(2\kappa-1)+2\kappa+1\right)=(2\kappa+1)\left(\frac{2\kappa^2+5\kappa+3}{3}\right)=\frac{1}{3}(2\kappa+1)(2\kappa+3)(\kappa+1)$$

Demak, (1) tasdiq n istalgan natural son bo‘lganda to‘g‘ri bo‘ladi.

2-mısol. Ayniyatni isbotlang:

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+\ldots+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3) \qquad (1)$$

Yechimi. 1. n=1 bo‘lsa, gipoteza to‘g‘ri bo‘ladi, chunki.

$$1\cdot 2\cdot 3=\frac{1}{4}\cdot 1\cdot(1+1)(1+2)(1+3)$$

2. n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaymiz, ya’ni.

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+\ldots+\kappa(\kappa+1)(\kappa+2)=\frac{1}{4}\kappa(\kappa+1)(\kappa+2)(\kappa+3)$$

tasdiqlashning n=k+1 uchun yani

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+\ldots+\kappa(\kappa+1)(\kappa+2)+(\kappa+1)(\kappa+2)(\kappa+3)=$$

$$=\frac{1}{4}\cdot(\kappa+1)(\kappa+2)(\kappa+3)(\kappa+4)$$

tengligining to‘g‘riligini isbotlaymiz:

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+\ldots+\kappa(\kappa+1)(\kappa+2)+(\kappa+1)(\kappa+2)(\kappa+3)=\frac{1}{4}\kappa(\kappa+1)(\kappa+2)(\kappa+3)+$$

$$+(\kappa+1)(\kappa+2)(\kappa+3)=(\kappa+1)(\kappa+2)(\kappa+3)\left(\frac{1}{4}\kappa+1\right)=\frac{1}{4}\cdot(\kappa+1)(\kappa+2)(\kappa+3)(\kappa+4)$$

Demak, (1) tasdiq n istalgan natural son bo‘lganda to‘g‘ri bo‘ladi.

3-mısol. Ayniyatni isbotlang:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1} \qquad (1)$$

Yechimi. 1. n=1 bo‘lsa, gipoteza to‘g‘ri bo‘ladi, chunki.

$$\frac{1}{1\cdot 2}=\frac{1}{1+1}$$

2. n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaymiz, ya’ni:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{\kappa(\kappa+1)}=\frac{\kappa}{\kappa+1}$$

tasdiqlashningń n=k+1 uchun yani

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{\kappa(\kappa+1)}+\frac{1}{(\kappa+1)(\kappa+2)}=\frac{\kappa+1}{\kappa+2}$$

tengligining to‘g‘riligini isbotlaymiz:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{\kappa(\kappa+1)}+\frac{1}{(\kappa+1)(\kappa+2)}=\frac{\kappa}{\kappa+1}+\frac{1}{(\kappa+1)(\kappa+2)}=\frac{1}{\kappa+1}\left(\kappa+\frac{1}{\kappa+2}\right)=$$

$$=\frac{1}{\kappa+1}\left(\frac{\kappa^2+2\kappa+1}{\kappa+2}\right)=\frac{1}{\kappa+1}\cdot\frac{(\kappa+1)^2}{\kappa+2}=\frac{\kappa+1}{\kappa+2}$$

Demak, (1) tasdiq n istalgan natural son bo‘lganda to‘g‘ri bo‘ladi.

4-mısol. Ayniyatni isbotlang:

$$\frac{x+1}{2}+\frac{x+3}{4}+\frac{x+7}{8}+...+\frac{x+2^n-1}{2^n}=\frac{(x-1)(2^n-1)}{2^n}+n \qquad (1)$$

Yechimi. 1. n=1 bo‘lsa, gipoteza to‘g‘ri bo‘ladi, chunki

$$\frac{x+1}{2}=\frac{(x-1)(2-1)}{2}+1$$

2. n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaymiz, ya’ni:

$$\frac{x+1}{2}+\frac{x+3}{4}+\frac{x+7}{8}+...+\frac{x+2^\kappa-1}{2^\kappa}=\frac{(x-1)(2^\kappa-1)}{2^\kappa}+\kappa$$

tasdiqlashning n=k+1 uchun yani

$$\frac{x+1}{2}+\frac{x+3}{4}+\frac{x+7}{8}+...+\frac{x+2^\kappa-1}{2^\kappa}+\frac{x+2^{\kappa+1}-1}{2^{\kappa+1}}=\frac{(x-1)(2^{\kappa+1}-1)}{2^{\kappa+1}}+\kappa+1$$

tengligining to‘g‘riligini isbotlaymiz:

$$\frac{x+1}{2}+\frac{x+3}{4}+\frac{x+7}{8}+\ldots+\frac{x+2^\kappa-1}{2^\kappa}+\frac{x+2^{\kappa+1}-1}{2^{\kappa+1}}=\frac{(x-1)(2^\kappa-1)}{2^\kappa}+\kappa+\frac{x+2^{\kappa+1}-1}{2^{\kappa+1}}=$$

$$=\frac{2\cdot(x-1)(2^\kappa-1)+(x-1)+2^{\kappa+1}}{2^{\kappa+1}}+\kappa=\frac{(x-1)(2^{\kappa+1}-2+1)}{2^{\kappa+1}}+\frac{2^{\kappa+1}}{2^{\kappa+1}}+\kappa=\frac{(x-1)(2^{\kappa+1}-1)}{2^{\kappa+1}}+\kappa+1$$

Demak, (1) tasdiq n istalgan natural son bo‘lganda to‘g‘ri bo‘ladi.

5-mısol. Ayniyatni isbotlang:

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\ldots+\frac{1}{n(n+1)(n+2)(n+3)}=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(n+1)(n+2)(n+3)}\right)$$

Yechimi. 1. n=1 bo‘lsa, gipoteza to‘g‘ri bo‘ladi, chunki

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{2\cdot 3\cdot 4}\right)=\frac{1}{3}\cdot\frac{3}{2\cdot 3\cdot 4}$$

2. n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaymiz, ya’ni:

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\ldots+\frac{1}{\kappa(\kappa+1)(\kappa+2)(\kappa+3)}=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(\kappa+1)(\kappa+2)(\kappa+3)}\right)$$

tasdiqlashningń n=k+1 uchun yani

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\ldots+\frac{1}{\kappa(\kappa+1)(\kappa+2)(\kappa+3)}+\frac{1}{(\kappa+1)(\kappa+2)(\kappa+3)(\kappa+4)}=$$
$$=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(\kappa+2)(\kappa+3)(\kappa+4)}\right)$$

tengligining to‘g‘riligini isbotlaymiz::

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\ldots+\frac{1}{\kappa(\kappa+1)(\kappa+2)(\kappa+3)}+\frac{1}{(\kappa+1)(\kappa+2)(\kappa+3)(\kappa+4)}=$$
$$=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(\kappa+1)(\kappa+2)(\kappa+3)}\right)+\frac{1}{(\kappa+1)(\kappa+2)(\kappa+3)}=$$
$$=\left(\frac{1}{6}-\frac{1}{(\kappa+2)(\kappa+3)(\kappa+4)}\right).$$

Demak, (1) tasdiq n istalgan natural son bo‘lganda to‘g‘ri bo‘ladi.

6-mısal. Ayniyatni isbotlang:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots-\frac{1}{2n}=\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n} \qquad (1)$$

Yechimi. 1. n=1 bo‘lsa, gipoteza to‘g‘ri bo‘ladi, chunki $1-\frac{1}{2}=\frac{1}{2}$

1. n=k bo‘lganda ham, gipoteza to‘g‘ri bo‘ladi deb hisoblaylik, ya’ni.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots-\frac{1}{2k}=\frac{1}{k+1}+\frac{1}{k+2}+\ldots+\frac{1}{2k} \qquad (2)$$

tasdiqlashningń n=k+1 uchun yani

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{2k-1}-\frac{1}{2k}+\frac{1}{2k+1}-\frac{1}{2(k+1)}=$$
$$=\frac{1}{k+2}+\ldots+\frac{1}{2k}+\frac{1}{2k+1}+\frac{1}{2(k+1)} \qquad (3)$$

tengligining to‘g‘riligini isbotlaymiz:

(3)-tengliktan (2)- tenglikning chap tomonidan chap tomonini va o‘ng tomonidan o‘ng tomonini tomonini olib, quyidagi tenglikni hosil qilamiz:

$$\frac{1}{2k+1}-\frac{1}{2(k+1)}=\frac{1}{2k+1}+\frac{1}{2(k+1)}-\frac{1}{k+1}$$

Demak, (2) tenglik to‘g‘ri bo‘lsa, (3) tenglikda ham to‘g‘ri bo‘ladi, bundan (1) tenglik

barcha natural sonlar uchun to‘g‘riligi kelib chiqadi.

XULOSA

Matematik induksiya metodi matematikaning asosiy isbotlash usullaridan biri bo‘lib, u ko‘plab formulalar, ayniyatlar va teoremalarni umumiy ko‘rinishda isbotlash imkonini beradi.

Ushbu metodning mohiyati shundan iboratki, mulohazaning eng kichik natural son uchun to‘g‘riligi tekshiriladi va agar u ma’lum bir son uchun to‘g‘ri bo‘lsa, undan keyingi son uchun ham amal qilishi isbotlanadi. Shu tarzda mulohaza barcha natural sonlar uchun isbotlangan hisoblanadi.

Matematik induksiya yordamida ayniyatlarni isbotlash o‘quvchilarda mantiqiy fikrlashni rivojlantirish, matematik isbotlash madaniyatini shakllantirish, hamda analitik tafakkurni kuchaytirishda katta ahamiyatga ega. Shuningdek, u matematik formulalar va qonuniyatlarni mustahkamlash, nazariy bilimlarni amaliyotda qo‘llash imkoniyatlarini kengaytiradi.

Foydalanilgan adabiyotlar

1. Э.Н.Балаян. “Новый репетитор по математике” - Ростов на Дону: “Феникс,” 2018.

2. В.Н.Литвиненко, А.Г.Мордкович. “Практикум по элементарной математике” - Москва: “Просвещение,” 1991.